

1. Isothermal Sound Waves

The first time someone tried to derive the behavior of sound waves, it was assumed that they behave isothermally, rather than adiabatically. Thus, we would use $dT/dt = 0$, rather than $d\theta/dt = 0$. Making this change, we would arrive at

$$\begin{aligned} \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} &= 0 \\ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + p \frac{\partial u}{\partial x} &= 0 \end{aligned} \tag{1a,b}$$

(a) Show that an acceptable basic state is $u = \bar{u}$, $p = \bar{p}$, $\rho = \bar{\rho}$, where all three fields are constants.

(b) Assume the isothermal sound waves are given by perturbations about this basic state: $u = \bar{u} + u'$, $p = \bar{p} + p'$, $\rho = \bar{\rho} + \rho'$. Substitute these forms for u , p , and ρ into (1a,b) and obtain the corresponding two linearized equations for the perturbation fields.

(c) If we eliminate variables in the two linearized equations to get a single equation in p' , we then have

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) \left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) p' - \frac{\bar{p}}{\bar{\rho}} \frac{\partial^2 p'}{\partial x^2} = 0 \tag{2}$$

Assume a wave solution, $p' = A \exp\{ik(x - ct)\}$, and derive the relationship for phase speed c in terms of \bar{u} , \bar{p} , and $\bar{\rho}$.

(d) What is the component of the group velocity in the x direction?

(e) Are isothermal sound waves faster or slower than true (adiabatic) sound waves? Why do you think that is the case?